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Fakultät für Informatik
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Winter term 2019/20
Assignment 14
January 27, 2020

# Efficient Algorithms and Data Structures I

*Deadline: February 3, 10:15 am in the **Efficient Algorithms** mailbox.*

## Homework 1 (4 Points)

Let $G = (V, E)$ be a directed graph with unit capacities, and suppose that for each node $v$ the number of edges into $v$ is equal to the number of edges out of $v$. That is, for all $v \in V$

$$|\{(u, v) \mid (u, v) \in E\}| = |\{(v, w) \mid (v, w) \in E\}|.$$

Let $x$, $y$ be two nodes of $G$, and suppose that the capacity of the minimum $x - y$-cut is $k$. Prove or disprove: The minimum $y - x$-cut also has capacity $k$.

## Homework 2 (4 Points)

Given $s$ and $t$ in a network $G = (V, E)$, we want to identify a minimum $(s, t)$-cut $A \subset V$ that minimizes the number of edges leaving $A$ among all minimum $(s, t)$-cuts. The edges in the network have integer capacities.
Show how to rescale the capacities of $G$ such that a single max-flow/min-cut computation yields the desired cut.

## Homework 3 (6 Points)

The ghost Ambrosius plans to simultaneously spook each of the $\ell$ floors of the FMI building in order to celebrate Richard finally leaving the department. For the big party, he needs $r_j$ ghosts for floor $F_j$.
Ambrosius must enlist ghosts from local haunted mansions for help. The ghosts living in mansions $M_1, \ldots, M_t$ are experienced. The ghosts living in mansions $M_{t+1}, \ldots, M_k$ are inexperienced. There are $u_i$ ghosts living in mansion $M_i$. A ghost from mansion $M_i$ will spook floor $j$ for $c_{ij}$ Euros.
Ambrosius knows that he needs at least one experienced ghost per floor. He wants to spend as little money as possible.

(a) Show how to formulate the above problem as a Minimum-Cost Flow Problem. Explain the different elements of your construction. Make sure to specify what a flow unit represents.

(b) Given an integral minimum cost flow in your network, show how to obtain an assignment of the ghosts to the floors.

## Homework 4 (6 Points)

In another dimension, $n$ goblin tribes are in trade with each other. For each tribe $i$, the value $s_i$ is its budget balance: negative $s_i$ indicates a deficit, while positive $s_i$ indicates a surplus. Tribe $i$ exports goods of value $e(i, j) \geq 0$ to tribe $j$.

When electing a new goblin king, the goblins follow a peculiar system. A subset $S$ of tribes is electable, if the sum of the budget surpluses of the tribes in $S$, minus the total value of all exports from tribes in $S$ to tribes not in $S$ is nonnegative. If there exists an electable set of tribes, the new goblin king is chosen at random from the members of all electable tribes. If there is no electable set of tribes, no king is chosen.

Model this problem as a minimum $s$-$t$-cut problem. Precisely describe the network you use. Show how a cut in your network corresponds to an electable set of tribes and vice versa.

## Bonus Homework 1 (6 Bonus Points)

FAQ Time! Please send your questions to your tutor via email at least 48 hours before the tutorial.

You may earn bonus points for asking good questions.

## Tutorial Exercise 1

A hospital needs to assign a group of $n$ neurosurgeons to $n$ patients. Each surgeon proposes, in decreasing order of preference, a list of three patients that he or she would like to perform surgery on. We want to determine whether there exists a *satisfiable assignment* (one that assigns the surgeons to the patients so that each surgeons obtains a patient on his or her list). If some satisfiable assignment is possible, we want to find the assignment that maximizes the number of surgeons with their most preferred patient, and further, among such assignments, the assignment that maximizes the number of patients with their second most preferred patient. Show how to solve this problem by solving a single assignment problem.

Be eccentric, narcissistic, and attract attention in innocent ways.

- J. Edmonds