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Assignment 13
January 21, 2019

Efficient Algorithms and Data Structures I

Deadline: January 28, 2019, 10:15 am in the ***Efficient Algorithms*** *mailbox.*

Homework 1 (4 Points)

Prove the following statements about maximum flow networks.

(a) If all edge capacities are even integers, then the maximum flow value is an even integer.

(b) Let e be an edge that belongs to some minimum cut. Show that any maximum flow saturates the edge e.

Homework 2 (5 Points)

Let $G = (V, E)$ be a network with two vertices $s, t \in V$. We call two $s-t$-paths *edge-disjoint* if they do not share an edge. Two $s-t$-paths are vertex-disjoint if they have no vertices in common other than s and t.
Prove or disprove the following statements

(a) There are $k > 1$ pairwise edge-disjoint $s-t$-paths in the network if and only if after deleting any $k-1$ edges, there still exists a path from s to t.

(b) There are $k > 1$ pairwise vertex-disjoint $s-t$-paths in the network if and only if after deleting any $k-1$ vertices, there still exists a path from s to t.

Homework 3 (5 Points)

We say that a bipartite graph $G = (V, E)$, where $V = L \cup R$, is d-regular if every vertex $v \in V$ has degree exactly d. Every d-regular bipartite graph has $|L| = |R|$. Prove that every d-regular bipartite graph has a matching of cardinality $|L|$ by arguing that a minimum cut of the corresponding flow network has capacity $|L|$.

Homework 4 (6 Points)

The ghost Ambrosius plans to simultaneously spook each of the ℓ floors of the FMI building in order to celebrate 50 years of computer science in Munich. For the big party, he needs r_j ghosts for floor F_j.
Ambrosius must enlist ghosts from local haunted mansions for help. The ghosts living in mansions $M_1, \dots, M_t$ are *experienced*. The ghosts living in mansions $M_{t+1}, \dots, M_k$ are *inexperienced*. There are u_i ghosts living in mansion M_i. A ghost from mansion M_i will spook floor j for c_{ij} Euros.

Ambrosius knows that he needs at least one experienced ghost per floor. He wants to spend as little money as possible.

(a) Show how to formulate the above problem as a Minimum-Cost Flow Problem. Explain the different elements of your construction. Make sure to specify what a flow unit represents.

(b) Given an integral minimum cost flow in your network, show how to obtain an assignment of the ghosts to the floors.

Tutorial Exercise 1

A shipping company wants to phase out a fleet of s (homogeneous) cargo ships over a period of p years. Its objective is to maximize its cash assets at the end of the p years by considering the possibility of prematurely selling ships and temporarily replacing them by charter ships.
The company faces a known nonincreasing demand for ships. Let d_k denote the demand of ships in year k. Each ship earns a revenue of r_k units in period k. At the beginning of year k, the company can sell any ship that it owns, accruing a cash inflow of s_k dollars. If the company does not own sufficiently many ships to meet its demand, it must hire additional charter ships. Let h_k denote the cost of hiring a ship for the kth year.
The shipping company wants to meet its commitments and at the same time maximize the cash assets at the end of the pth year.
Model this problem as a minimum cost flow problem!

Tutorial Exercise 2

In the famous *bin packing* problem, we are given n items of weights $a_1, a_2, \ldots, a_n$ and we are asked to pack them into as few bins as possible. Each bin can hold at most weight 1 and the items are not splittable.
Show that the bin packing problem can be solved by transforming it into a matching problem if $1/3 < a_j < 1$ for each $j = 1, \ldots, n$.