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Problem set 9
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Summer Semester 2016

Online and Approximation Algorithms

Due June 17, 2016 before 10:00

Exercise 1 (k-Server on a Line - 10 points)

Consider the k-server problem where all servers and requests are located on a continuous straight line. Algorithm DC (*Double Coverage*) serves each incoming request for point x as follows.

If x is on the left of all servers, move the closest server to it. Treat the case where x is on the right of all servers similarly. Otherwise x is located between two servers s_i and s_j. Move both servers with equal speed towards x until one of them reaches x (i.e., if s_i is the closest, then both servers move distance $d(s_i, x)$).

Let $s_1, s_2, \ldots, s_k$ and $a_1, a_2, \ldots, a_k$ be the locations of the servers by DC and OPT, respectively. We define the potential function $\Phi = k \cdot M + D$, where M is the minimum cost perfect matching in the bipartite graph between $s_1, s_2, \ldots, s_k$ and $a_1, a_2, \ldots, a_k$, while $D = \sum_{i<j} d(s_i, s_j)$ is the sum of all pairwise distances between the servers of DC.

(a) Show that Φ satisfies the following properties:

(i) If the adversary's cost increases by y, then the change in the potential is $\Delta\Phi \leq ky$.

(ii) If the cost of DC increases by y', then the change in the potential is $\Delta\Phi \leq -y'$.

(b) Show that DC is k-competitive.

Exercise 2 (2-Server Algorithm - 10 points)

Consider the following 2-server algorithm. After serving a request, label the server at the request s_1 and the other server s_2 (if both servers are at the request, break ties arbitrarily). Consider the next request r and set $b = d(s_1, r)$. If $d(s_2, r) < 3b$, serve r with s_2. Otherwise, serve r with s_1 and also move s_2 a distance $3b$ towards r. Prove that this algorithm is $O(1)$-competitive in any Euclidean space.

Exercise 3 (Max Cut - 10 points)

In the lecture, a deterministic $\frac{1}{2}$-approximation algorithm for the Max Cut problem was given.

Consider the following randomized algorithm to solve Max Cut. Each vertex is randomly and indepentdently assigned a value 0 or 1. All vertices with value 1 are in S and all vertices with value 0 are in $V \setminus S$. Prove that the approximation ratio of this algorithm is also $\frac{1}{2}$.

Exercise 4 (Eulerian Cycle - 10 points)

Recall that a Eulerian cycle is a cycle in a graph that visits every edge exactly once. Show that a connected graph $G = (V, E)$, with possibly multiple edges between a pair of vertices, contains a Eulerian cycle if and only if every vertex $v \in V$ is of even degree.