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Problem set 8
June 3, 2016
Summer Semester 2016

Online and Approximation Algorithms

Due June 10, 2016 before 10:00

Exercise 1 (Lower envelope - 10 points)

Recall the Lower Envelope Algorithm (LEA) from the lecture. In the lecture we showed, that the algorithm is $3 - 2\sqrt{2}$ competitive for general state systems.
We consider the special case where the costs for powering down are additive, i.e., for $i < j < k$, powering down from state s_i to s_j and from s_j to s_k is equally expensive as powering down from s_i to s_k.
Prove that in this setting LEA is 2-competitive.
Hint: Fold the cost for powering down into the cost for powering up. This yields a system where powering down is free and only transitions from low-power states to higher-power states create costs.

Exercise 2 (Power-Down Mechanisms - 10 points)

Recall the energy efficiency problem presented in class in which there is a system with an active state s_0 and several lower-power sleep states $s_1, s_2, \ldots, s_\ell$. Each state s_i has an individual power-consumption rate r_i and there is a cost $d_{i,j}$ for transitioning from state s_i to state s_j where the transition costs satisfy the triangle inequality. For a maximal idle period of length t, we denote by $OPT(t)$ the optimal offline cost of this period. Show that $OPT(t)$ is a continuous and concave function of t.

Exercise 3 (Randomized Search and One-way trading - 10 points)

We consider one-way trading problems and randomized search algorithms. Recall the following theorem:

Theorem 1 *(i) Let ALG_1 be any randomized one-way trading algorithm. Then there exists a deterministic one-way trading algorithm ALG_2 such that for any price sequence σ, $ALG_2(\sigma) = E[ALG_1(\sigma)]$.*
(ii) Let ALG_2 be any deterministic one-way trading algorithm. Then there exists a randomized search algorithm ALG_1 such that $E[ALG_1(\sigma)] = ALG_2(\sigma)$.

Show the following statements.

(a) The competitive ratio of any one-way trading algorithm is independent of transaction costs determined by a fixed percentage applied to the amount spent. In this case, the equivalence of Theorem 1 obviously holds.

(b) When we introduce transaction fees determined by a fixed percentage applied to the prices rates, the competitive ratio improves, but Theorem 1 still holds. **NB: There is a mistake in this exercise. Please check the solution for more details.**

(c) When fixed transaction costs are introduced, the deterministic competitive ratio increases and there is no longer an equivalence between deterministic one-way trading algorithms and randomized search algorithms.

Exercise 4 (EXPO - 10 points)

Recall the online search problem presented in class and the EXPO algorithm for solving it. Moreover, let μ be a probability distribution of the natural numbers $\mathbb{N}$ according to which the number i is chosen with probability q_i. Now, consider the EXPO(μ) algorithm which is defined as follows: Choose the price $p \cdot 2^i$ with probability q_i, where p is the first price revealed.
Show that EXPO(μ) is $\frac{2}{q_j}$-competitive, for some $j \in \mathbb{N}$.