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Problem set 1
April 15, 2016
Summer Semester 2016

Online and Approximation Algorithms

Due April 22, 2016 at 08:15, in class

Exercise 1 (Ski Rental - 10 points)

The ski rental problem is defined as follows: Assume that renting a pair of skis costs 1 per day while buying a pair of skis costs b. Every day we have to decide, in an online fashion, whether we want to continue renting skis for another day or buy a pair of skis. At some unknown time D, we will break our leg and have to quit skiing. Our goal is to minimize the cost of skiing.

(a) What is the optimal offline cost?

(b) Develop a $(2 - \frac{1}{b})$-competitive online algorithm ALG for the ski rental problem and prove its competitiveness.

Exercise 2 (List Scheduling on Unrelated Machines - 10 points)

In class, an online greedy algorithm for the problem of scheduling n jobs on m identical machines was presented. In this exercise, we consider the same problem in a more complex environment with m *unrelated* machines $M_1, M_2, \ldots, M_m$.
In this setting, the processing time of a job J_j, $1 \leq j \leq n$, depends on the machine on which it is executed. Specifically, if job J_j is executed by machine M_i, then its processing time is $p_{i,j}$. The greedy algorithm presented in class for identical machines can be easily extended for unrelated machines as follows: Schedule each job J_j on a machine M_i that results in the schedule with minimum makespan. Prove that the competitive ratio of Greedy is not smaller than $\dfrac{m}{1+\varepsilon}$, where ε is an arbitrary small constant.

Hint: There exists an example where each job has finite processing time only on two machines and infinite processing time on all the other machines.

Exercise 3 (First-in First-out - 10 points)

Recall that FIFO is the online paging algorithm that evicts the page that has been in fast memory for the longest time. Prove that FIFO is k-competitive, where k is the number of pages that fit in fast memory.

Exercise 4 (Demand Paging - 10 points)

Paging algorithms that do not evict pages unless there is a page fault are called *demand paging*. Prove that any paging algorithm can be modified to be demand paging without increasing the overall number of memory replacements on any request sequence.